hep-th/9611089
CAT/95-06

On the spontaneous break down of massive gravities in 2+1 dimensions

C. Aragone[a,*], **P.J.Arias**[b,c,†] **and A.Khoudeir**[c,‡]

[a] *Departamento de Física, Universidad Simón Bolívar, Caracas, Venezuela.*

[b] *Grupo de Campos y Partículas, Departamento de Física, Facultad de Ciencias, Universidad Central de Venezuela, AP 47270, Caracas 1041-A, Venezuela.*

[c] *Centro de Astrofísica Teórica, Facultad de Ciencias, Universidad de los Andes, AP 26, La Hechicera, Mérida 5101, Venezuela.*

Abstract

We show that locally Lorentz invariant, third order, topological massive gravity can not be broken down neither to the local diffeomorphism subgroup nor to the rigid Poincaré group. On the other hand, the recently formulated, locally diffeomorphism invariant, second order massive triadic (translational) Chern-Simons gravity breaks down on rigid Minkowski space to a double massive spin-two system. This flat double massive action is the uniform spin-two generalization of the Maxwell-Chern-Simons-Proca system which one is left with after U(1) abelian gauge invariance breaks down in the presence of a sextic Higgs potential.

[*] **In Memoriam**
[†] email: parias@tierra.ciens.ucv.ve
[‡] adel@ciens.ula.ve

Topological massive gravity [1] remains a puzzling theory. In spite of being third order it is yet unclear whether it is renormalizable [2]. In addition, although several types of exact solutions have been found [3], none of them are proper Chern-Simons(CS) black holes (or black strings [4]). The black holes recently found by Bañados, Teitelboim and Zanelli [5] are three-dimensional pure Einstein black holes. They are not specific solutions of topological massive gravity.

Another aspect which up to now did not receive much attention concerns about the possible symmetry breaking process the theory can undergo. This letter aims to investigate this problem. We study the possibility of breaking down its local Lorentz invariance and analyze the physical relevance of the two reasonable models that in such case one is left with.

Since for both types of process the answer will be negative, we then explore whether topological triadic CS-gravity [6] might be spontaneously broken (to Minkowski space). In this case the answer is positive.

The first process we study arises when one assumes that local Lorentz invariance is lost by the addition of the diffeomorphism invariant triadic CS term. This action has, then, three constituents: the original third order Lorentz-CS term $\sim \omega\varepsilon\partial\omega$, minus the second order Einstein Kinetic term $\sim \omega_{pa}\omega^{ap}$, plus the first order triadic CS term $\sim e\varepsilon\partial e$ which breaks the local Lorentz symmetry, since it only possess diffeomorphism invariance. In principle, this action has a good aspect since it is possible to show it is a pure spin-2 action which contains two massive excitations of opposite helicities or two massive excitations with the same helicity (according to the relative sign of the typical massess of the two different CS terms). We shall see that, however, the energy of this system is not definite positive. Consequently this system has not physical relevance.

Since this process is not allowed we then investigate whether if one takes a more simple minded point of view and fully breaks local Lorentz invariance by the addition of an algebraic Fierz-Pauli massive term and goes down to flat space one obtains a meaningful pure spin-2 action.

We shall see that also in this case the broken system has a pure spin-2 content which might have either one or three physical poles for its cubic propagator, according to the relative value of the two masses involved in the original action. Disregarding the case of one physical excitation (it implies two additional complex poles) we then analyse the case of the likely existence of the three different positive masses. Also in this case the system shows unbounded energy. Consequently it has no physical significance either.

We conclude that Lorentz-CS topological massive gravity cannot be spontaneously broken. (We do not foresee any reason why a mixture of the triadic-CS term with the Fierz-Pauli one would provide a positive answer).

In view of these negative results for the posibility of breaking down standard topological gravity we examine massive vector (triadic) CS-gravity [6]. This is a curved theory propagating one massive spin-2 excitation.

We show that in this case, the addition of the Fierz-Pauli (metric) mass term to the linearized action (composed by the sum of the three dimensional Einstein action $\sim \omega\varepsilon\partial e - \omega^2$ and the triadic CS term $\sim e\varepsilon\partial e$) gives rise to a physically relevant pure spin-2 theory on flat Minkowski space which propagates two spin-2 massive excitations of opposite helicities and different masses. Triadic CS topological gravity can be broken down to Miskowski space while Lorentz CS theory does not allow this type of process.

Now we come to the definitions in order to be able to present the technical details.

Three dimensional Einstein action enters in all the theories we consider.Its first order form is

$$E\cdot \equiv \kappa^{-2} < {\omega_p}^a \varepsilon^{prs} \partial_r e_{sa} - 2^{-1} {e_p}^a \varepsilon^{prs} \varepsilon_{abc} {\omega_r}^b {\omega_s}^c > \tag{1}$$

where κ, in units of $[m]^{-1/2}$, is the three dimensional gravitational constant. ${e_p}^a$ and ${\omega_p}^a$ constitute the basic triadic fields and connections. Middle of the alphabet letters $p, r, \cdots$ name world indices while $a, b, \cdots$ denote local Lorentz ones, $\varepsilon^{012} = +1$ and the flat metric η_{ab} has positive signature ($\eta_{00} = -1$). As all the remaining elementary actions we will introduce in this paper, it is locally trivial, i.e. the associated field equations tell us that this action does not propagate local physical excitations.

A simple and useful property of this action is how it looks like when one works in a second order formulation. After introducing in eq.(1) ${\omega_p}^a$ in terms of the triadic fields e as given by the ω-field equations

$$\varepsilon^{pqr} (\partial_q e_{ra} - {\omega_q}^c \varepsilon_{cba} {e_r}^b) = 0, \tag{2}$$

one is lead to the second order form of E. It turns out to be

$$E = 2^{-1} \kappa^{-2} < \varepsilon^{prs} \varepsilon_{abc} {e_p}^a {\omega_r}^b(e) \omega_s^c(e) >, \tag{3}$$

which in the linearized case, takes the typical Fierz-Pauli form $\sim 2^{-1} \omega_{pa} \omega_{ap} - 2^{-1} \omega^2$ in terms of the (now non independent) variables ω. Topological massive gravity (which from now on we call Lorentz-CS gravity) needs the presence of another locally trivial action: the third order, locally conformal and Lorentz invariant Chern-Simons term

$$L\cdot \equiv (2\mu_1 \kappa^{-2})^{-1} < {\omega_p}^a \varepsilon^{prs} \partial_r \omega_{sa} - 2(3)^{-1} \varepsilon^{prs} \varepsilon_{abc} {\omega_p}^a {\omega_p}^b {\omega_s}^c > \tag{4}$$

where $\omega = \omega(e)$ as given by eq.(2). Its action is $L - E$.

Triadic CS-gravity [6] on the other hand is defined by adding to E the first order, diffeomorphism invariant triadic-CS term:

$$T\cdot \equiv 2^{-1} \mu_2 \kappa^{-2} < {e_p}^a \varepsilon^{prs} \partial_r e_{sa} > . \tag{5}$$

The associated full second order action is $E + T$.

We want to investigate whether Lorentz-CS gravity can be broken by a term $\sim$ T; i.e. we wonder whether $L - E + T$ is a satisfactory spin-2 theory having a mass spectra corresponding to some spliting of the initial mass μ in two different (but closely related) masses.

A reasonable insight on the physical significance of this proposal can be obtained by analysing the behaviour of the associated third-order linearized theory on flat Minkowski space. We start considering its first order action S_0. It is the quadratic part of $L - E + T$ when we expand ${e_p}^a = \delta_p^a + \kappa h_p^a$ in terms of κ

$$\begin{aligned} S_0\cdot = (2\mu)^{-1} < {\omega_p}^a \varepsilon^{prs} \partial_r \omega_{sa} > -2^{-1} < \omega_{pa} \omega^{ap} - \omega^2 > -2^{-1} m < {h_p}^a \varepsilon^{prs} \partial_r h_{sa} > \\ + < {\lambda_p}^a \varepsilon^{prs} (\partial_r h_{sa} - {\omega_r}^b \varepsilon_{bsa}) > . \end{aligned} \tag{6}$$

Its equivalent third order version arises from introducing the values of $\omega = \omega(h)$ (obtained from variations of the λ's) into S_0.

Independent variations of ω, h, λ yield the triplet of field equations (FE).

$$E^p{}_a\cdot = \mu^{-1}\varepsilon^{prs}\partial_r\omega_{sa} - \omega_a{}^p + \delta_a{}^p\omega - \lambda_a{}^p + \delta_a{}^p\lambda = 0, \tag{7}$$

$$F^p{}_a = -m\varepsilon^{prs}\partial_r h_{sa} + \varepsilon^{prs}\partial_r\lambda_s{}^a = 0, \tag{8}$$

and

$$G^p{}_a\cdot = \varepsilon^{prs}\partial_r h_{sa} - \omega_a{}^p + \delta_a{}^p\omega = 0. \tag{9}$$

Considering the lower spin sector of these eqs., i.e. computing $E\cdot \equiv E^p{}_p$, F, G, $\partial_p E^p{}_a, \cdots$ and $\varepsilon_{pab}E^{pa} \equiv \cdot\check{E}_b$, $\check{F}_b$, $\check{C}_b$ it is straightforward to see that this system only propagates spin-2 excitations.

Both, the spin-1 $\varepsilon_{pab}\omega^{pa}, \cdots, \varepsilon_{pab}\lambda^{pa}, \partial_p\omega_{pa}, \partial_p\lambda_{pa}$ and the scalar sector of ω, h, λ vanish in the harmonic gauge $\partial_p h_{pa} = 0$.

Projection of the FE (7) (8) (9) upon the spin-2^+ (spin-2^-) subspaces using the pseudo-seudospin-$2^\pm$ projectors [7], gives

$$(X-1)\omega^{T+} - \lambda^{T+} = 0 \quad , \quad -mXh^{T+} + X\lambda^{T+} = 0 \quad , \quad \omega^{T+} = Xh^{T+} \tag{10}$$

where $X = \mu^{-1}\Box^{1/2}, \mu = 1$. m means the dimensionless relation $m\mu^{-1}$ and h^{T+} denotes the spin-2^+ part of h_{pa}.

The inverse propagator is therefore

$$\Delta^+(X) = X[X(X-1) - m]. \tag{11}$$

There is a positive mass $m = 2^{-1} + (4^{-1}+m)^{1/2}$ in the spin-2^+ sector. Similarly, since $\Delta^-(X) = X[X(X+1) - m]$ we might have a spin-2^- excitation with mass $m^- = -2^{-1} + (4^{-1}+m)^{1/2}$.

We want to see whether this system has its energy bounded from below (or not). It will be shown that, independently of the sign of m, the light-front (LF) generator is unbounded and consequently action (6) is physically meaningless, in spite of the fact that, from a covariant point of view, the system (7), (8), (9), seems to propagate two spin-2 decoupled excitations.

In order to have this, we calculate the value of the LF-generator of action (6) in terms of its two unconstrained variables ω_{vv} and λ_{vv}. Light front coordinates (u, v) are defined by

$$\eta^{11} = 1 = -\eta^{uv}, \quad u\cdot = 2^{-1/2}(x^0 - x^2), \quad v\cdot = 2^{-1/2}(x^0 + x^2), \quad \varepsilon^{1vu} = +1. \tag{12}$$

Time derivatives are written $\partial_u f = \dot{f}$ and the LF-spacelike ones are denoted $\partial_v f = f'$

One starts from the covariant expressions (6) of S_0 and express this action in terms of the 27 LF-field components $\omega_{uu} \equiv \cdot\omega_u$, ω_{uv}, ω_{vu}, $\omega_v\cdot \equiv \omega_{vv}$, $\omega_1\cdot \equiv \omega_{11}$, ω_{1u}, ω_{u1}, ω_{1v} and $\omega_{v1}, \cdots, \lambda_u, \lambda_{uv}, \cdots, \lambda_{1v}, \lambda_{v1}$.

It is inmediate to realize that $\omega_{ua}, h_{ub}, \lambda_{uc}$ are multipliers associatted with nine differential constraint equations which can be solved, providing the values of $\omega_{1a}, h_{1b}, \lambda_{1c}$ as functions of the remaining nine intermediate variables $\omega_{va}, h_{vb}, \lambda_{vc}$. Their solution is:

$$\widehat{\omega}_{1v} = (\partial_1 + 1)\widehat{\omega}_v + \widehat{\lambda}_v \quad , \quad \widehat{h}_{1v} = \partial_1\widehat{h}_v + \widehat{\omega}_v, \tag{13a, b}$$

$$\widehat{\lambda}_{1v} = \partial_1\widehat{\lambda}_v + m\widehat{\omega}_v, \tag{13c}$$

$$\omega_1 = \partial_1 \widehat{\omega}_{v1} + \widehat{\omega}_{1v} + \widehat{\lambda}_{1v} \quad , \quad h_1 = \partial_1 \widehat{h}_{v1} + \widehat{\omega}_{1v}, \tag{14a,b}$$

$$\lambda_1 = \partial_1 \widehat{\lambda}_{v1} + m\widehat{\omega}_{1v}, \tag{14c}$$

$$\omega_{1u}{}' = (\partial_1 - 1)\omega_{vu} - \lambda_{vu} + \partial_1 \widehat{\omega}_{v1} + \partial_1 \widehat{\lambda}_{v1} + (m+1)\widehat{\omega}_{1v} + \widehat{\lambda}_{1v} \tag{15a}$$

$$h_{1u}{}' = \partial_1 h_{vu} - \omega_{vu} + \partial_1 \widehat{\omega}_{v1} + \widehat{\omega}_{1v} + \widehat{\lambda}_{1v} \tag{15b}$$

$$\lambda_{1u}{}' = \partial_1 \lambda_{vu} - m\omega_{vu} + m\partial_1 \widehat{\omega}_{v1} + m\widehat{\omega}_{1v} + m\widehat{\lambda}_{1v} \tag{15c}$$

where we introduced redefinitions like $\omega_{v1} \equiv \cdot\widehat{\omega}'_{v1}$, $\omega_v \equiv \cdot\widehat{\omega}''_v$, $\omega_{1v} \equiv \cdot\widehat{\omega}'_{1v}$ for the three sets of variables ω, λ, h.

In principle, the intermediate expression of S_0 obtained in terms of the nine intermediate variables $\omega_{va}, h_{vb}, \lambda_{vc}$ might have $\omega_{vu}, h_{vu}, \lambda_{vu}$ in the dynamical germ (the piece of $S_0 \sim p\dot{q}$).

However it turns out after using eqs. (13), (14) that these three variables are not present in this part of the action. While ω_{vu} and λ_{vu} constitute two additional Lagrange multipliers, h_{vu} has totally disappeared.

Independent variations of $\omega_{vu}, \lambda_{vu}$ lead to the final two differential constraints of S_0. Their solution shows the symmetry of the $1v$-components $\omega_{1v}, \lambda_{1v}$, i.e.

$$\widehat{\omega}_{v1} = \widehat{\omega}_{1v} \quad , \quad \widehat{\lambda}_{v1} = \widehat{\lambda}_{1v} \tag{16a,b}$$

Now it is immediate to obtain the unconstrained form of the evolution generator G of action $S_0 \sim p\dot{q} - G$. Since, at the initial stage when one writes down S_0 in terms of the LF-variables, G had the form:

$$G =< (\widehat{\omega}_{v1} + \widehat{\lambda}_{v1})\omega_{1v}{}' + \widehat{\omega}_{v1}\lambda_{1u}{}' >; \tag{17}$$

it is straightforward to realize that, after insertion of the values (15) of $\omega_{1u}{}', \lambda_{1u}{}'$ in it, G becomes:

$$G =< [(m+1)\widehat{\omega}_{1v} + \widehat{\lambda}_{1v}]^2 - m^2\widehat{\omega}^2_{1v} > . \tag{18}$$

This explicitly shows that the generator is a non semidefinite positive quadratic expression. Consequently the unconstrained reduced form of S_0, even written in terms of the unique two gauge-invariant variables $\widehat{\lambda}_v, \widehat{\omega}_v$, does not have physical relevance. One can say that the presence of both types of CS terms is inconsistent. This situation is peculiar of Lorentz-CS gravity (there is no analogous third order CS theory for vector fields).

Since LCS-gravity can not be broken through a triadic CS type of term we now consider the possibility of a harder type of breaking induced by the presence, in flat Minkowsky space, of a Fierz-Pauli mass term. In order to investigate this possibility we examine the linearized system. It consists of

$$S_3 \cdot \equiv L^Q - E^Q + 2^{-1}m^2\varepsilon < h_{pa}h^{ap} - h^2 > \tag{19}$$

where L^Q, and E^Q are the quadratic parts (in terms of κ) of the exact curved actions (4) and (1) respectively, $e_{pa} \equiv \eta_{pa} + \kappa h_{pa}$.

It is convenient to start from a first order system equivalent to (19). It reads ($\omega \to \kappa\omega$)

$$\begin{aligned} S_1 \cdot = (2\mu)^{-1} < \omega_p{}^a \varepsilon^{prs}\partial_r \omega_{sa} > -2^{-1} < \omega_p{}^a \omega_a{}^p - \omega^2 > \\ +2^{-1}m^2\varepsilon < h_{pa}h^{ap} - h^2 > + < \lambda_p \varepsilon^{prs}(\partial_r h_s{}^a - \omega_r{}^b \varepsilon_{bsa}) > \end{aligned} \tag{20}$$

where λ, ω, h are three sets of independent variables and $\varepsilon = \pm 1$.

Calculations are still simpler using new dimensionless variables $x^r_{new} = mx^r$, $h^{pa} = m^{1/2} h^{pa}_{new}$, $\omega^{pa} = m^{3/2}\omega^{pa}_{new}$, $\lambda^{pa} = m^{3/2}\lambda^{pa}_{new}$. (Without risk of ambiguity the subscript new is pressumed in all variables from now on).

Independent variations with respect to them yield three sets of field equations(FE)

$$\delta S_1/\delta\omega \rightarrow E^p{}_a = 0 \quad , \quad \delta S_1/\delta h \rightarrow F^p{}_a = 0 \quad , \quad \delta S_1/\delta\lambda \rightarrow G^p{}_a = 0. \tag{20a, b, c}$$

Straightforward calculations with $E^p{}_p, \partial_p E^p{}_a, \partial^a\partial_p E^p{}_a, \partial^a\partial_p E^p{}_a, \varepsilon^{pba}E_{pa}$ show that all lower spin variables contained in $\omega_p{}^a, h_p{}^a, \lambda_p{}^a$ vanish on the FE. (There is no need of decomposing triadic variables into their irreducible symmetric and antisymmetric parts). Then, the system generated by action (20) is a pure spin-2 system.

In the presence of a consistent current j_{pa} ($\varepsilon^{pab}j_{pa} = 0$, $\partial_p j_{pa} = 0$) the triplet of FE reads

$$E^{T\overline{pa}} = \frac{1}{2\mu}(\varepsilon_{p.}{}^{rs}\partial_r\omega^T_{\overline{sa}} + \varepsilon_a{}^{rs}\partial_r\omega^T_{\overline{sp}}) - \omega^T_{\overline{pa}} - \lambda^T_{\overline{pa}} = 0, \tag{21}$$

$$F^{T\overline{pa}} = \frac{1}{2}(\varepsilon_{p.}{}^{rs}\partial_r\lambda^T_{\overline{sa}} + \varepsilon_a{}^{rs}\partial_r\lambda^T_{\overline{sp}}) - \varepsilon h^T_{\overline{pa}} = -j^T_{pa}, \tag{22}$$

$$G^{T\overline{pa}} = \frac{1}{2}(\varepsilon_{p.}{}^{rs}\partial_r h^T_{\overline{sa}} + \varepsilon_a{}^{rs}\partial_r h^T_{\overline{sp}}) - \omega^T_{\overline{pa}} = 0. \tag{23}$$

in terms of the symmetric traceless transverse parts ($h^T_{\overline{pp}} = 0 = \partial_p h^T_{\overline{pa}}$) of the initial non-symmetric variables $\omega_p{}^a, h_p{}^a, \lambda_p{}^a$.

The last step in this quick preliminary covariant analysis consists in using the simple helicity projectors $P_2^{\pm}$ [7] for separating, given a symmetric traceless, transverse second order rank tensor, its two spin-2^+ and spin-2^- components.

For second rank symmetric traceless transverse tensors $h^T_{\overline{pa}}$ we have

$$h^{T+}_{\overline{pa}} - h^{T-}_{\overline{pa}}\cdot = \{(P_2^+ - P_2^-)h^T\}_{\overline{pa}} = 2^{-1}\Box^{-\frac{1}{2}}(\varepsilon_{p.}{}^{rs}\partial_r h^T_{\overline{sa}} + \varepsilon_a{}^{rs}\partial_r h^T_{\overline{sp}}), \tag{24}$$

$$h^{T+}_{\overline{pa}} + h^{T-}_{\overline{pa}} = \{(P_2^+ + P_2^-)h^T\}_{\overline{pa}} = h^T_{\overline{pa}}. \tag{25}$$

Projection of eqs. (21)$\cdots$(23) (on the spin-2^+ sector) leads to ($\Box^{1/2} = X$)

$$Xh^{T+} = \omega^{T+} \quad , \quad \mu^{-1}X\omega^{T+} - \omega^{T+} = \lambda^{T+} \quad , \quad X\lambda^{T+} + \varepsilon h^{T+} = -j^{T+}. \qquad 26(a, b, c)$$

It is immediate to calculate the inverse propagator of this system. It is

$$\Delta^+(X)\cdot = X^2(\frac{X}{\mu} - 1) + \varepsilon. \tag{27}$$

($\Delta^-(X) = \Delta^+(-\mu, X)$).The mass spectrum of our system are the positive solutions of $\Delta^+(X)$ and $\Delta^-(X)$. The analysis of this cubic equation shows that, for $\mu > 0$, the more interesting case will be when $\varepsilon = +1$. In this case we might have two positive masses of helicity $s = 2^+$ and one positive mass for the $s = 2^-$ excitation. (When $\varepsilon = -1$ we have complex poles in addition to only one positive mass).

We explain now why even in what seems to be the more interesting case, $\varepsilon = +1$, the system is unphysical, the reason being that its energy is unbounded.

The proof consists in obtaining the unconstrained action in terms of the three spin-2 independent variables this system has, and then looking at the explicit form of the energy. We perform the reduction process in the LF coordinates, see eqs(12).The initial system (20) has $3\times 9 = 27$ independent variables. Again for simplicity in the case of two equal indices we skip one of them, i.e. $h_{uu} = h_u, h_{vv} = h_v, h_{11} = h_1$. In terms of these LF variables action (20) looks like

$$S_1 = \mu^{-1} < \omega_{1u}\dot{\omega}_v + \omega_{1v}\dot{\omega}_{vu} - \omega_1\dot{\omega}_{v1} > + < h_{1u}\dot{\lambda}_v + h_{1v}\dot{\lambda}_{vu} - h_1\dot{\lambda}_{v1} > +$$

$$+ < \lambda_{1u}\dot{h}_v + \lambda_{1v}\dot{h}_{vu} - \lambda_1\dot{h}_{v1} > + < \omega_u\mathcal{C}^u + \omega_{uv}\mathcal{C}^v + \omega_{u1}\mathcal{C}^1 > +$$

$$+ < h_{ua}\mathcal{D}^a > + < \lambda_{ub}\mathcal{E}^b + < \omega_{v1}\omega_{1u} - \omega_1\omega_{vu} > +$$

$$+\varepsilon < h_1 h_{vu} - h_{v1}h_{1u} > + < \lambda_{v1}\omega_{1u} - \lambda_1\omega_{vu} + \lambda_{1u}\omega_{v1} - \omega_1\lambda_{vu} > \tag{28}$$

where we explicitly see the presence of, at least, 9 differential constraints $\mathcal{C}^a, \mathcal{D}^a, \mathcal{E}^a = 0$ associated with the 9 Lagrange multipliers $\omega_{ua}, h_{ua}, \lambda_{ua}$. They allow to obtain $\omega_{1a}, h_{1a}, \lambda_{1a}$ in terms of the nine variables $\omega_{va}, h_{va}, \lambda_{va}$. The latter set contains more variables than the ones we would expect for a system that describes three independent massive degrees of freedom in 2+1 dimensions.

It turns out to be convenient to introduce the new variables: $\widehat{\lambda}_v, \widehat{\omega}_v, \widehat{h}_v, \widehat{\lambda}_{v1}, \widehat{\omega}_{v1}, \widehat{h}_{v1}, \widehat{\lambda}_{1v}, \widehat{\omega}_{1v}, \widehat{h}_{1v}$ defined by

$$\widehat{\lambda}_v{}''\cdot \equiv \lambda_v \quad , \quad \widehat{\lambda}_{v1}{}'\cdot \equiv \lambda_{v1} \quad , \quad \widehat{\lambda}_{2v}{}'\cdot \equiv \lambda_{1v} \quad , \quad \cdots \tag{29}$$

and similar ones exchanging λ for h, ω. As usually happens in light-front coordinates the 9 constraints are linear ordinary differential equations in the “space-like” variable v. They can be solved in terms of $\widehat{\lambda}_v, \widehat{\lambda}_{v1}, \lambda_{vu}, \widehat{\omega}_v, \widehat{\omega}_{v1}, \omega_{vu}, \widehat{h}_v, \widehat{h}_{v1}$ and h_{vu}

$$\widehat{\omega}_{1v} = (\partial_1 + \mu)\widehat{\omega}_v + \mu\widehat{\lambda}_v \quad , \quad \omega_{1u}{}' = (\partial_1 - \mu)\omega_{vu} - \mu\lambda_{vu} + \mu\omega_1 + \mu\lambda_1 \tag{30a, b}$$

$$\omega_1 = \partial_1\widehat{\omega}_{v1} + \mu\widehat{\omega}_{1v} + \mu\widehat{\lambda}_{1v} \quad , \tag{30c}$$

$$\widehat{\lambda}_{1v} = \partial_1\widehat{\lambda}_v - \varepsilon_p\widehat{h}_v \quad , \quad \lambda_{1v}{}' = \partial_1\lambda_{vu} - \varepsilon_p h_1 + \varepsilon_p h_{vu} \quad , \tag{31a, b}$$

$$\lambda_1 = \partial_1\widehat{\lambda}_{v1} - \varepsilon_p\widehat{h}1v \quad , \tag{31c}$$

$$\widehat{h}_{1v} = \partial_1\widehat{h}_v - \widehat{\omega}_v \quad , \quad h_{1u}{}' = \partial_1 h_{vu} - \omega_1 - \omega_{vu} \quad , \tag{32a, b}$$

$$h_1 = \partial_1\widehat{h}_{v1} - \widehat{\omega}1v \quad . \tag{32c}$$

Insertion of these values of $\lambda_{1a}, \omega_{1b}, h_{1c}$ into S_1 yields, in principle, an apparently unconstrained functional in terms of $\lambda_{va}, \omega_{vb}, h_{vc}$, as expected.

Something interesting happens. S_1 has the form $\sim p\dot{q} - G$. The dynamical germ (the part of $S_1 \sim p\dot{q}$) does not depend upon $h_{vu}, \omega_{vu}, \lambda_{vu}$. These variables only appear linearly in the light-front generator G of the system. We have found the remaining three Lagrange multipliers of the system. Independent variations with respect to them provide the three additional constraints $\mathcal{H}_h, \mathcal{H}_\omega, \mathcal{H}_\lambda$ needed in order to have a final S_1 fully unconstrained. Consequently S_1 can be cast as a functional of the three independent physical variables $\widehat{\lambda}_v, \widehat{\omega}_v, \widehat{h}_v$,

$$\mathcal{H}_h \sim \delta S_1/\delta h_{vu} = 0 \to \widehat{\omega}_{v1} = -\varepsilon\mu\widehat{\lambda}_v - \varepsilon(\partial_1 + \mu)\widehat{\omega}_v = -\varepsilon\widehat{\omega}_{1v}, \tag{33}$$

$$\mathcal{H}_\omega \sim \delta S_1/\delta\omega_{vu} = 0 \rightarrow \widehat{h}_{v1} = -\varepsilon\widehat{\omega}_v - \varepsilon\partial_1\widehat{h}_v = -\varepsilon\widehat{h}_{1v}, \tag{34}$$

$$\mathcal{H}_\lambda \sim \delta S_1/\delta\lambda_{vu} = 0 \rightarrow \widehat{\lambda}_{v1} = \omega_h\lambda_{1v} + (1+\varepsilon)\widehat{\omega}_{1v}. \tag{35}$$

The final step is to introduce these values in S_1. In turns out that the unconstrained light-front generator has the diagonal, non definite positive form

$$< G >=< \widehat{\omega}_{1v}^2 + [\mu(\widehat{\omega}_{1v} + \widehat{\lambda}_{1v}) - \varepsilon\widehat{h}_{1v}]^2 - \varepsilon^2\widehat{h}_{1v}^2 > . \tag{36}$$

The system, having an unbounded light-front-generator, does not have physical interest.

Thinking in terms of our initial Lorentz CS gravity, we can say that this theory can not be broken down by a Fierz-Pauli mass term.

It is worth pointing out what happens if we do not have the Fierz-Pauli-mass term. This is linearized Lorentz CS standard topological gravity. Following along the same lines, one finds that $\widehat{\omega}_{1v} = 0$, $\varepsilon \rightarrow 0$, and the generator becomes the non negative expression

$$< G >_{LCS}^{lim}= \mu^2 < \widehat{\lambda}_{1v}^2 > . \tag{37}$$

In this case the constraints tell us that $\widehat{\lambda}_{1v} = \partial_1\widehat{\lambda}_v$ and, futhermore that $\widehat{\lambda}_v = -(\mu^{-1}\partial_1 + 1)\widehat{\omega}_v$. Choosing $h = 2^{1/2}\partial_1\widehat{\omega}_v$ as the basic dynamical variable the unconstrained form of linearized LCS gravity becomes

$$S_{LCS}^{lim} =< h'\dot{h} > -2^{-1} < h[-\partial_1^2 + \mu^2]h >, \tag{38}$$

as expected.

Now we apply the same methods to investigate the physical relevance of spontaneously breaking triadic CS topological gravity whose exact $S_{TCS} = E + T$ action was recently analyzed [6].

The symmetry breaking process we imagine leads to consider, on flat Minkowski space, the quadratic system defined by

$$S_2 :=< {\omega_p}^a\varepsilon^{prs}\partial_r h_{sa} > -2^{-1} < \omega_{pa}\omega^{ap} - \omega^2 > +2^{-1}\mu < {h_p}^a\varepsilon^{prs}\partial_r h_{sa} >$$
$$-2^{-1}m^2 < h_{pa}h^{ap} - h^2 > . \tag{39}$$

Although this is the first order form of the system, independent variations of ω lead to its quadratic second order expressions in terms of the weak field h_{pa}.

Independent variations of h, ω lead to the set of two field equations:

$$\delta S/\delta {h_p}^a \rightarrow {E^p}_a\cdot = \varepsilon^{prs}\partial_r\omega_{sa} + \mu\varepsilon^{prs}\partial_r h_{sa} - m^2({h_a}^p - \ {\delta_a}^p h) = 0, \tag{40}$$

$$\delta S/\delta {\omega_p}^a \rightarrow {F^p}_a\cdot = \varepsilon^{prs}\partial_r h_{sa} - {\omega_a}^p + {\delta_a}^p\omega = 0. \tag{41}$$

It is straightforward to see this system has a pure spin-2 content.

Taking traces ${E^p}_p, {F^p}_p$, antisymmetric parts $\varepsilon_{pab}E^{pa}$, $\varepsilon_{pab}F^{pa}$ and divergences $\partial_p {E^p}_a, \partial_p {F^p}_a$ of the field equations one is lead to observe the vanishing of all the scalar and vector parts of h_{pa}, ω_{pa} ($h = \omega = 0, \varepsilon_{pab}h^{pa} = \varepsilon_{pab}\omega^{pa} = 0$, $\partial_p h_{pa} = 0 = \partial_p\omega_{pa}$).

There is no need of using the standard symmetric and antisymmetric components of h_{pa}, ω_{pa}. It is unconvenient. The symmetric parts of equations (40), (41) have the form:

$$X(h^{T+} - h^{T-}) = \omega^{T+} + \omega^{T-}, \tag{42}$$

$$X(\omega^{T+} - \omega^{T-}) + \mu X(h^{T+} - h^{T-}) = h^{T+} + h^{T-}, \tag{43}$$

where we are using dimensionless variables ($\sim m = 1$). The characteristic equations for each spin-$2^{\pm}$ component are $\Delta_{\pm}(X) = X^2 \pm \mu X - 1$. The system has positive masses $m^{\pm} = \mp 2^{-1}\mu + (1 + 4^{-1}\mu^2)^{1/2}$ for the respective h^{T+} and h^{T-} spin-2 excitations.

The viability of this system will emerge from considering the associated unconstrained action and showing that its energy is bounded from below.

We shall see this is true. S_2 is a physical theory. Indeed it is the peculiar CS splitting of the mass degeneracy of double massive, three dimensional Einstein-Fierz-Pauli action.

In terms of the light-front variables S_2 takes the initial form

$$S_2 =< h_{1u}\dot{\omega}_v + h_{1v}\dot{\omega}_{vu} - h_1\dot{\omega}_{v1} > + < (\omega_{1u} + \mu h_{1u})\dot{h}_v + (\omega_{1v} + \mu h_{1v})\dot{h}_{vu} - (\omega_1 + \mu h_1)\dot{h}_{v1} >$$

$$+ < \omega_{v1}\omega_{1u} + h_{v1}h_{1u} - \omega_1\omega_{vu} - h_1 h_{vu} > +$$
$$+ < \omega_u \mathcal{C}^u + \omega_{uv}\mathcal{C}^v + \omega_{u1}\mathcal{C}^1 + h_u\mathcal{D}^u + h_{uv}\mathcal{D}^v + h_{u1}\mathcal{D}^1 >, \tag{44}$$

where $\mathcal{C}^a, \mathcal{D}^b$ constitute the initial set of six differential constraints associated with the accompanying Lagrange-multipliers ω_{ua}, h_{ub}. The role of these constraints is to provide the values of ω_{1a}, h_{1a} in terms of the seemingly more fundamental components ω_{va}, h_{vb}. Once they are solved, instead of the initial expression of S_2 in terms of eighteen covariant variables ω_{pa}, h_{qb} we still have a reduced version in terms of, at most, six independent elements: ω_{va}, h_{vb}.

It is, again, convenient to introduce $\widehat{\omega}_v, \widehat{\omega}_{v1}, \widehat{\omega}_{1v}$

$$\widehat{\omega}_v{''}\cdot \equiv \omega_v \quad , \quad \widehat{\omega}_{v1}{'}\cdot \equiv \omega_{v1} \quad , \quad \widehat{\omega}_{1v}{'}\cdot \equiv \omega_{1v}, \tag{45}$$

and similar ones $\widehat{h}_v, \widehat{h}_{v1}, \widehat{h}_{1v}$ for the basic spin-2 field.

In terms of them, one can solve $\mathcal{C}^a = 0 = \mathcal{D}^b$ since they constitute a set of the ordinary linear differential equations in the LF-"space-like" variable v. Doing so one obtains

$$\mathcal{C}^a = 0 \sim \widehat{h}_{1v} = \widehat{\omega}_v + \partial_1\widehat{h}_v \quad , \quad h_{1u}{'} = \partial_1 h_{vu} + \omega_1 - \omega_{vu} \quad , \quad h_1 = \widehat{\omega}_{1v} + \partial_1\widehat{h}_{v1}, \tag{46a, b, c}$$

and

$$\mathcal{D}^b = 0 \sim \widehat{\omega}_{1v} = (\partial_1\mu)\widehat{\omega}_v + \widehat{h}_v \quad , \quad \omega_1 = \partial_1\widehat{\omega}_{v1} + (-\mu\partial_1 + \mu^2 + 1)\widehat{\omega}_v + (\partial_1 - \mu)\widehat{h}_v,$$
$$\omega_{1u}{'} = \partial_1\omega_{1v} + (\mu\partial_1 - 1)h_{vu} + h_1 - \mu h_{1u}{'}. \tag{47a, b, c}$$

The covariant analysis suggest that the system has two physical relevant independent (spin-$2^{\pm}$) excitations. Six are too many.

The interesting point is that there are two additional Lagrange multipliers in the intermediate reduced form of $S_2(\omega_{va}, h_{vb})$. Neither of ω_{uv}, h_{vu} appear in the dynamical germ $S_2(\omega_{va} h_{vb})$, but they are only present, linearly, in the LF-generator of the dynamical evolution, i.e.

$$G =< \widehat{\omega}_{v1}\omega_{1u}{'}|_{\omega_{vu}=0=h_{vu}} + \widehat{h}_{v1}h_{1u}{'}|_{\omega_{vu}=0=h_{vu}} + \omega_{vu}\mathcal{H}_a + h_{vu}\mathcal{H}_h > . \tag{48}$$

It turns out that the solution of these last two differential constraints $\mathcal{H}_\omega = 0 = \mathcal{H}_h$ is rather simple and can be cast in the form

$$\widehat{\omega}_{v1} = \widehat{\omega}_{1v} \quad , \quad \widehat{h}_{v1} = \widehat{h}_{1v}. \tag{49a, b}$$

Since $\widehat{\omega}_{1v}\widehat{h}_{1v}$ are given in terms of $\widehat{\omega}_v, \widehat{h}_v$, insertion of the information contained in eqs. (49a,b) leads to the final unconstrained form of S_2, expressed in terms of the two independent variables ω_v, h_v. One gets for S_2 (we abandon hats notation)

$$
\begin{gathered}
S_2(\omega_v, h_v) =< -\omega_v 2(1+\mu^2)\omega_v{}'^{\cdot} - 2h_v h_v'^{\cdot} + 4\mu h_v \omega_v'^{\cdot} - \\
-\{h_v(1+\mu^2-\partial_1^2)h_v + \omega_v[(1+\mu^2)^2+\mu^2-(1+\mu^2)\partial_1^2]\omega_v + 2h_v\mu[\partial_1^2-\mu^2-2]\omega_v\} > . \qquad (50)
\end{gathered}
$$

This quadratic expression can be written in a simpler way if one shifts $h_v \to \widetilde{h}_v = h_v - \mu\omega_v$. S_2 becomes the almost diagonal expression (forgetting the tilde in $\widetilde{h}$)

$$
S_2(\omega_v, h_v) =< 2\omega_v'\dot{\omega_v} + 2h_v'\dot{h_v} - \{h_v(1-\partial_1^2)h_v + \omega_v(-\partial_1^2)\omega_v + (\omega_v - \mu h_v)^2\} > . \qquad (51)
$$

We observe that the LF-generator is clearly non negative. Also, as a final check, independent variactions of ω_v, h_v yield a system of two coupled equations, whose inverse propagator can be quickly computed. It turns out to be (we have set the Fierz-Pauli mass equal to 1)

$$
\Delta(\omega_v, h_v) = \Box^2 - (\mu^2+2)\Box + 1 \equiv \Delta_+(X)\Delta_-(X) \qquad (52)
$$

In terms of the mass $X = \Box^{1/2}$ we introduced before. This quartic polynominal has two positive $m^{\pm} = \mp 2^{-1}\mu + (1+4^{-1}\mu^2)^{1/2}$ and two additional negative roots $-m^{\pm}$.

Our original covariant action $S_2 = E + T - FP$ represents a physical system of two decoupled spin-2 physical excitations having different masses m^+, m^- whose LF-"energy" is bounded. (Of course, the same results arise if one performs a canonical $2+1$ analysis of S_2 [8]).

In terms of the two initial masses of S_2 (eq.39), $m^{\pm}$ are determined by

$$
m^{\pm} = \mp 2^{-1}\mu + (4^{-1}\mu^2 + m^2)^{\frac{1}{2}} \qquad (53)
$$

Considering the system as the result of having spontaneously broken down the exact, curved TCS curved gravity, (which has one excitation of mass μ) we observe that this spin-2 situation is exactly the uniform generalization of the spontaneous breakdown of the vector case [6]. The consequence of the presence of the Fierz-Pauli mass term is the creation of the new spin-2 excitation having the smaller mass $-2^{-1}|\mu| + (4^{-1}\mu^2 + m^2)^{1/2}$ while the previously existing excitation of mass $|\mu|$ has been shifted to $2^{-1}|\mu| + (4^{-1}\mu^2 + m^2)^{1/2}$.

We conclude emphasizing the much different behaviour of LCS and TCS gravities. The former cannot be spontaneously broken by the presence of neither a TCS-term nor a Fierz-Pauli one, while the already Lorentz broken, diffeomorphism invariant, $E+T$ action can still be broken down one step further by addition of a Fierz-Pauli mass term.

REFERENCES

1. S. Deser and R. Jackiw Ann. Phys. **140** (1982) 372; (E) **185** (1988) 406.
2. S. Deser and Z. Yang, Class. Quantum Grav. **7** (1990) 1603.
3. I. Vuorio, Phys. Lett. **B163** (1985) 91; R. Percacci, P. Sodano and I. Vuorio, Ann. Phys. **176** (1987) 344; C. Aragone, Class. Quantum Grav. **4** (1987) L1; G. Clément, Class. Quantum Grav.**9** (1992) 35.
4. J. Horne and G. Horowitz, Nucl. Phys. **B368** (1992) 444.
5. M Bañados, C. Teitelboin and J. Zanelli, Phys. Rev. Lett. **69** (1992) 1849.
6. C. Aragone, P. J. Arias and A. Khoudeir, Il Nuovo Cimento **109B** (1994) 303.
7. C. Aragone and A. Khoudeir, Phys. Lett. **B173** (1986) 141.
8. C. Aragone, P. J. Arias and A. Khoudeir, Proceedings Silarg 7, edited by M. Rosenbaum *et al*, World Scientific (Singapure) 1991 p 437.